1. Introduction and results. The classical Schwarz reflection principle states that a continuous map $f$ between real-analytic curves $M$ and $M'$ in $\mathbb{C}$ that locally extends holomorphically to one side of $M$, extends also holomorphically to a neighborhood of $M$ in $\mathbb{C}$. It is well-known that the higher-dimensional analog of this statement for maps $f: M \to M'$ between real-analytic CR-submanifolds $M \subset \mathbb{C}^N$ and $M' \subset \mathbb{C}^{N'}$ does not hold without additional assumptions (unless $M$ and $M'$ are totally real). In this paper, we assume that $f$ is $C^1$-smooth and that the target $M'$ is real-algebraic, i.e. contained in a real-algebraic subset of the same dimension. If $f$ is known to be locally holomorphically extendible to one side of $M$ (when $M$ is a hypersurface) or to a wedge with edge $M$ (when $M$ is a generic submanifold of higher codimension), then $f$ automatically satisfies the tangential Cauchy-Riemann equations, i.e. it is CR. On the other hand, if $M$ is minimal, any CR-map $f: M \to M'$ locally extends holomorphically to a wedge with edge $M$ by Tumanov’s theorem [Tu88] and hence, in that case, the extension assumption can be replaced by assuming $f$ to be CR.

Local holomorphic extension of a CR-map $f: M \to M'$ may clearly fail when $M'$ contains an (irreducible) complex-analytic subvariety $E'$ of positive dimension and $f(M) \subset E'$. Indeed, any nonextendible CR-function on $M$ composed with a nontrivial holomorphic map from a disc in $\mathbb{C}$ into $E'$ yields a counterexample. Our first result shows that this is essentially the only exception. Denote by $E'$ the set of all points $p' \in M'$ through which there exist irreducible complex-analytic subvarieties of $M'$ of positive dimension. We prove:

**Theorem 1.1.** Let $M \subset \mathbb{C}^N$ and $M' \subset \mathbb{C}^{N'}$ be respectively connected real-analytic and real-algebraic CR-submanifolds. Assume that $M$ is minimal at a point $p \in M$. Then for any $C^\infty$-smooth CR-map $f: M \to M'$, at least one of the following conditions holds:

(i) $f$ extends holomorphically to a neighborhood of $p$ in $\mathbb{C}^N$;

(ii) $f$ sends a neighborhood of $p$ in $M$ into $E'$.

If $M'$ is a real-analytic hypersurface, the set $E'$ consists exactly of those points that are not of finite type in the sense of D’Angelo [D’A82] (see Lempert [L86] for the proof) and, in particular, $E'$ is closed. The same fact also holds if $M'$ is any real-analytic submanifold or even any real-analytic subvariety (see [D’A91]). However, in general, $E'$ may not even be a real-analytic subset (see Example 2.1). In case $E' = V'$ is a subvariety, we have:

**Corollary 1.2.** Let $M \subset \mathbb{C}^N$ and $M' \subset \mathbb{C}^{N'}$ be as in Theorem 1.1. Assume that $M$ is minimal at a point $p \in M$ and that all positive-dimensional irreducible complex-algebraic subvarieties in $M'$ are contained in a fixed (complex-algebraic) subvariety $V' \subset \mathbb{C}^{N'}$.

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