Course 321 2006-07

S h e e t 2

Due: after the lecture next Thursday

Exercise 1

Assuming that $x_n \to x$, $y_n \to y$ as $n \to \infty$ in a metric space (X, d), show that $d(x_n, y_n) \to d(x, y)$.

Exercise 2

When do you have equality in Hölder's inequality

(i) in $\mathbb{R}^n$?

(ii) in its integral form?

(Hint. Check for equalities in the proof.)

Exercise 3

Is the following true:

$$\bigcup_{1 \le p < \infty} l^p = c_0?$$

Exercise 4

Show that $\lim_{p \mapsto \infty} \|x\|_p = \|x\|_\infty$

(i) for $x \in \mathbb{R}^n$;

(ii) for $x \in l^{p_0}$ for some $1 \le p_0 < \infty$.

(Hint. Reduce to the case $\|x\| \le 1$.)

Exercise 5

Let μ be the measure on $\mathbb{N}$, associating to any subset $A \subset \mathbb{N}$ the number of its elements (the counting measure). Show that l^p is isometric to $L^p(\mathbb{N}, \mu)$ for any $1 \le p \le \infty$.

Exercise 6

(i) Show that a normed space is non-separable if it has an uncountable set of disjoint balls of radius 1.

(ii) Does the same statement remain true without the radius restriction?

Exercise 7

Show that the space c (consisting of all convergent sequences with norm $\|\cdot\|_\infty$) is separable.

Exercise 8

For any interval $[a,b]$ $(-\infty < a < b < +\infty)$, show that $L^1[a,b] \neq L^2[a,b]$. Which space is larger?