Course 321 2006-07

Sheet 1

Due: after the lecture next Thursday

Exercise 1

Give an example of a binary relation on a set which is

(i) reflexive and symmetric, but not transitive;
(ii) reflexive and transitive, but not symmetric;
(iii) symmetric and transitive, but not reflexive;
(iv) transitive, but neither reflexive nor symmetric.

Exercise 2

Prove that the following sets have the cardinality of the continuum:

(i) any interval (a, b) with $-\infty \leq a < b \leq +\infty$;
(ii) any interval $[a, b)$ with a and b as above;
(iii) the set of all infinite sequences of integers.

Exercise 3

(i) Give an example of two (partial) order relations on a finite set that are nonisomorphic.
(ii) Is such an example possible for linear order relations?

Exercise 4

(i) Prove that the sum of ordinals is associative, i.e. $(\alpha_1 + \alpha_2) + \alpha_3 = \alpha_1 + (\alpha_2 + \alpha_3)$ holds for any ordinals α_1, α_2, α_3.
(ii) Is the product of ordinals also associative?
(ii) Is the product of ordinals commutative?

Exercise 5

Let $n \geq 1$ be the ordinal of the finite set $\{1, \ldots, n\}$ and ω be the ordinal of the set of all natural numbers. Prove or disprove:

(i) $n + \omega = \omega$;

(ii) $\omega + n = \omega$;

(iii) $\omega + n + \omega = \omega + \omega$;

(iv) $\omega + \omega = 2 \cdot \omega$;

(vi) $\omega + \omega = \omega \cdot 2$;

Exercise 6

Show that the set of all disks contained in the square $[0, 1] \times [0, 1]$, ordered by inclusion, contans infinitely many maximal elements but no largest element.