Course 212 2004-05

S h e e t 9

Due: after the lecture next Thursday

Exercise 1

Prove that if C, C' are compact subsets of a Hausdorff space T, then $C \cap C'$ and $C \cup C'$ are also compact.

Exercise 2

Which of the following subsets of $\mathbb{R}^2$ are (a) connected:

(a) the union of the open discs $B_1(x_1) \cup B_1(x_2)$ of radius 1, $x_1 = (1, 0)$, $x_2 = (-1, 0)$;

(b) the union of the closures of the same discs;

(c) the set of all points with at least one coordinate in $\mathbb{Q}$.

Exercise 3

Determine the connected components of the sets $\mathbb{Q}^2$, $\mathbb{R}^2 \setminus \mathbb{Q}^2$ and $(\mathbb{R} \setminus \mathbb{Q}) \times \mathbb{Q}$.