Course 212 2004-05

Sheet 8

Due: after the lecture next Thursday

Exercise 1

Determine whether $f\colon A \to \mathbb{R}$ is uniformly continuous on A, when

(a) $A = (0,1)$, $f(x) = x$;

(b) $A = (0,1)$, $f(x) = \mathsf{sin}(1/x)$;

(c) $A = [1,+\infty)$, $f(x) = 1/x$.

Exercise 2

Determine whether the family of functions $(f_\lambda)_{\lambda\in\mathbb{R}}$ is equicontinuous at the origin:

(a) $f_\lambda(x) = x + \lambda$;

(b) $f_\lambda(x) = (x+\lambda)^2$;

(c) $f_\lambda(x) = \lambda x$.

Exercise 3

Prove that if $A \subset \mathbb{R}$ is not compact, then there exists a continuous function $f\colon A \to \mathbb{R}$ which is not bounded on A. (Hint. Consider separately cases A is not bounded and A is not closed).

Exercise 4

Show that X is connected if and only if there does not exists a nonconstant continuous map f from X into the space $\{0,1\}$ with discrete topology.