Course 212 2004-05

Sheet 7

Due: after the lecture next Thursday

Exercise 1

Given topological spaces X, Y, Z and a point $x \in X$, prove that $Y \times Z$ is homeomorphic to the subspace $\{x\} \times Y \times Z$ of $X \times Y \times Z$.

Exercise 2

Let X be a Hausdorff topological space. Show that points (as one-point sets) are closed in X. Is the same true for general topological spaces?

Exercise 3

Prove that any topological space with finitely many points is compact. Is this true for countable topological spaces?

Exercise 4

Which of the following subsets of ${\rm I\!R}$ or ${\rm I\!R}^2$ are compact:

$$[0,1); \quad [0,+\infty); \quad \{(x,y): x^2+y^2=1\}; \quad \{(x,y): y \ge x^2\}.$$