

**Course 212 2004-05**

S h e e t 6

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Due: after the lecture next Thursday

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**Exercise 1**

What are the possible topologies on the set  $\{1, 2\}$ ?

**Exercise 2**

Prove that the boundary of a set in a topological space is always closed.

**Exercise 3**

Give an example of topologies  $\mathcal{T}_1, \mathcal{T}_2$  on a set  $X$  such that  $\mathcal{T}_1 \not\subset \mathcal{T}_2$  and  $\mathcal{T}_2 \not\subset \mathcal{T}_1$ .

**Exercise 4**

Let  $X$  be a topological space. Prove that  $f: X \rightarrow \mathbb{R}$  is continuous if and only if  $f^{-1}(-\infty, a)$  and  $f^{-1}(a, \infty)$  are open in  $X$ .