Exercise 1
Which of the following subspaces of $\mathbb{R}^n$ (for suitable $n$) are complete?

(i) $\{1/n : n \in \mathbb{N}\} \cup \{0\}$

(ii) $\mathbb{Q} \cap [-1, 1]$  

(iii) $\{(x, y) \in \mathbb{R}^2 : x > 0, y \geq 1/x\}$

Exercise 2
Show that a Cauchy sequence is convergent if and only if it has a convergent subsequence.

Exercise 3
Let $f: (0, 1/4) \to (0, 1/4)$ be given by $f(x) = x^2$. Show that $f$ is a contraction without a fixed point.

Exercise 4
Suppose that $f: M \to M$ is a self-map of a complete metric space $M$ such that for some integer $r$, the iterated map $f^{(r)} = f \circ f \circ \ldots \circ f$ ($r$ times) is a contraction. Prove that $f$ has a unique fixed point $p$ in $M$. 