November 29, 2003 Lecturer Dmitri Zaitsev Michaelmas Term 2004

Course 212 2004-05

Sheet 4

Due: after the lecture next Thursday

Exercise 1

Which of the following subspaces of $\mathbb{R}^n$ (for suitable n) are complete?

(i) $\{1/n : n \in \mathbb{N}\} \cup \{0\}$

(ii) $\mathbb{Q} \cap [-1, 1]$

(iii) $\{(x, y) \in \mathbb{R}^2 : x > 0, y \geq 1/x\}$

Exercise 2

Show that a Cauchy sequence is convergent if and only if it has a convergent subsequence.

Exercise 3

Let $f\colon (0, 1/4) \to (0, 1/4)$ be given by $f(x) = x^2$. Show that f is a contraction without a fixed point.

Exercise 4

Suppose that $f\colon M \to M$ is a self-map of a complete metric space M such that for some integer r, the iterated map $f^{(r)} = f \circ f \circ \ldots \circ f$ (r times) is a contraction. Prove that f has a unique fixed point p in M.