November 15, 2004 Lecturer Dmitri Zaitsev Michaelmas Term 2004

Course 212 2004-05

Sheet 3

Due: after the lecture next Thursday

Exercise 1

Given two continuous functions $f, g\colon M \to \mathbb{R}$ on a metric space M, show that the functions $|f|$ and $\min(f, g)$ are also continuous.

Exercise 2

Prove that, for any metric space M and any subset S of M, $\mathrm{Int}(\mathrm{Int}(S)) = \mathrm{Int}(S)$, where $\mathrm{Int}(S)$ is the interior of S.

Exercise 3

Let M be a metric space, and let S be a subset of M. If x is a limit point of S, show that each open ball $B_\varepsilon(x)$, $\varepsilon > 0$, contains an infinite number of distinct points of S.

Exercise 4

Find the boundary in $\mathbb{R}^2$ of the sets: (a) $S = \mathbb{Z} \times \mathbb{Z}$; (b) $S = \mathbb{Q} \times \mathbb{Z}$; (c) S is the graph of the function $y = \sin(1/x)$.