Course 212 2004-05

Sheet 10

Due: after the lecture next Thursday

Exercise 1

Consider the vector space V of all sequences $x = (x_n)$ of real numbers with $\lim x_n = 0$.

(a) Show that $\|x\| := \sup_n |x_n|$ is a norm on V.

(b) Is $(V, \|\cdot\|)$ a Banach space?

Exercise 2

Given two normed vector spaces V_1 and V_2, define a norm on the direct sum $V_1 \oplus V_2$ by $\|v_1 \oplus v_2\| := \|v_1\| + \|v_2\|$.

(a) Show that $V_1 \oplus V_2$ is a normed space.

(b) Assume that V_1 and V_2 are Banach, is it always true that $V_1 \oplus V_2$ is also Banach?

Exercise 3

Let V be the space $C[0,1]$ of all continuous functions (with the standard norm $\|\cdot\|_\infty$) and $K(x,y)$ a given continuous function on $[0,1]\times[0,1]$.

(a) Define a linear operator $A\colon V \to V$ by the formula

$$(Af)(x) := \int_0^1 K(x,y)\, f(y)\, dy$$

and show that A is a bounded linear operator.

(b) Calculate the norm of A in case $K(x,y) \geq 0$.

(c) Give an upper estimate for the norm of A for general $K(x,y)$.