

**Course 212 2004-05**

## S h e e t 1

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Due: after the lecture next Monday

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**Exercise 1**

Define  $f: \mathbb{R} \rightarrow \mathbb{R}$  as follows:  $f(x) = x$  for  $x$  rational and  $f(x) = 1$  for  $x$  irrational. At what points is  $f$  continuous? Justify your answer.

**Exercise 2**

Let  $A$  be a 3-point set. Give an example of a function  $d: A \times A \rightarrow \mathbb{R}$  satisfying (M1) and (M2) but not (M3). Is this possible for a 2-point set? Justify your answer.

**Exercise 3**

Let  $(A, d)$  be a metric space,  $a, b \in A$  be two different points and  $r := d(a, b)$ . Show that  $B_{r/2}(a) \cap B_{r/2}(b) = \emptyset$ , where  $B_s(a) := \{x \in A : d(x, a) < s\}$  denotes the open ball with center  $a$  and radius  $s$ .