February 29, 2020 Lecturer Dmitri Zaitsev Hilary Term 2020

MAU34206 - Harmonic Analysis I 2020

S h e e t 3

Due: after the lecture next Wednesday

Exercise 1

Compute the translations F_i, F_{-i}, F_{-1}, $(F_i)_{-i}$ for F given by $F(\zeta) := \frac{1}{\zeta}$, $\zeta \in \mathbb{T}$.

Exercise 2

Compute the convolution $F * \Phi$ on $\mathbb{T}$, where

$$F(\zeta) = \zeta^{-1}, \quad \Phi(\zeta) = (\zeta + 1)^2.$$

Exercise 3

Show that $C(\mathbb{T}) \subset L^2(\mathbb{T}) \subset L^1(\mathbb{T})$ and

(i) $\|F\|_2 \leq \|F\|_\infty$ for $F \in C(\mathbb{T})$.

(ii) $\|F\|_1 \leq \|F\|_2$ for $F \in L^2(\mathbb{T})$. Hint. For (ii), use the Cauchy-Schwarz inequality.

Exercise 4

Show that the Fourier transform of a convolution on $\mathbb{T}$ is the product of the Fourier transforms of the factors, i.e.

$$\widehat{F * G} = \widehat{F}\widehat{G}, \quad F, G \in L^1(\mathbb{T}).$$

Exercise 5

For $F \in L^1(\mathbb{T})$, shows that the $L^1(\mathbb{T})$-valued integral $\int_0^1 F_{e^{itx}}\, dx$ is independent of $t \in \mathbb{R}$.