Exercise 1
Compute the translations $F_i$, $F_{-i}$, $F_{-1}$, $(F_i)_{-i}$ for $F$ given by $F(\zeta) := \frac{1}{\zeta}$, $\zeta \in \mathbb{T}$.

Exercise 2
Compute the convolution $F \ast \Phi$ on $\mathbb{T}$, where
\[ F(\zeta) = \zeta^{-1}, \quad \Phi(\zeta) = (\zeta + 1)^2. \]

Exercise 3
Show that $C(\mathbb{T}) \subset L^2(\mathbb{T}) \subset L^1(\mathbb{T})$ and
(i) $\|F\|_2 \leq \|F\|_\infty$ for $F \in C(\mathbb{T})$.
(ii) $\|F\|_1 \leq \|F\|_2$ for $F \in L^2(\mathbb{T})$. Hint. For (ii), use the Cauchy-Schwarz inequality.

Exercise 4
Show that the Fourier transform of a convolution on $\mathbb{T}$ is the product of the Fourier transforms of the factors, i.e.
\[ \hat{F \ast G} = \hat{F} \hat{G}, \quad F, G \in L^1(\mathbb{T}). \]

Exercise 5
For $F \in L^1(\mathbb{T})$, shows that the $L^1(\mathbb{T})$-valued integral $\int_0^1 F e^{itx} \, dx$ is independent of $t \in \mathbb{R}$. 