February 17, 2020 Lecturer Dmitri Zaitsev Hilary Term 2020

MAU34206 - Harmonic Analysis I 2020

S h e e t 2

Due: after the lecture next Wednesday

Exercise 1

For the group $G = \mathbb{Z}_3$ and function

$$f([k]) = a_k, \quad k \in \{0, 1, 2\},$$

where $a_1, a_2, a_3 \in \mathbb{C}$ are given:

(i) determine the dual group $\widehat{G}$ (including the group operation);

(ii) compute the Fourier transform $\widehat{f}(\chi)$, $\chi \in \widehat{G}$.

Exercise 2

The same questions as in Exercise 1 for $G = \mathbb{Z}_2 \oplus \mathbb{Z}_2$, the direct sum of cyclic groups, and $f\colon G \to \mathbb{C}$ given by $f([0] \oplus [0]) = 1$ and 0 otherwise.

Exercise 3

For $f\colon [0, 1] \to \mathbb{C}$ given by

$$f(x) = \begin{cases} 1, & x < 1/2 \\ -1 & x \geq 1/2, \end{cases}$$

(i) compute the Fourier coefficients;

(ii) compute the Fourier partial sums $S_0 f$, $S_1 f$, $S_2 f$.