Exercise 1
Give a manifold structure on the $n$-torus $\mathbb{R}^n/\mathbb{Z}^n$.

Exercise 2
Show that the map
\[ t \in \mathbb{R} \mapsto (\cos(\sqrt{2}t)(2 + \cos t), \sin(\sqrt{2}t)(2 + \cos t), \sin t) \in \mathbb{R}^3 \]
defines an immersion whose image $S$ is not a submanifold of $\mathbb{R}^3$. Show that $S$ is dense in the torus
\[ \{(\cos(2 + \cos s), \sin(2 + \cos s), \sin s) \in \mathbb{R}^3 : s, t \in \mathbb{R}\} \].

Exercise 3
The Veronese and Segre maps are given respectively by
\[ v: (x, y) \in \mathbb{R}^2 \mapsto (x^n, x^{n-1}y, x^{n-2}y^2, \ldots, x^iy^j, \ldots, y^n) \in \mathbb{R}^{n+1} \]
and
\[ s: (x_1, \ldots, x_n, y_1, \ldots, y_m) \in \mathbb{R}^{m+n} \mapsto (x_iy_j)_{1 \leq i \leq n, 1 \leq j \leq m} \in \mathbb{R}^{mn} \].
Is the image of $v$ (resp. $s$) a submanifold? If not, what is the maximal open set $U$ in the target space such that the intersection of $U$ with image of $v$ (resp. the image of $s$) is a submanifold?

Exercise 4
A quadric $Q$ (or hyperquadric) in $\mathbb{R}^n$ is the subset given by $\sum_{ij} a_{ij}x^ix^j + \sum_k b_kx^k + c = 0$, where the symmetric matrix $(a_{ij})$ is invertible. Show that $Q$ is a submanifold everywhere except possibly one point. What is its dimension?

Exercise 5
Let $X$ be a $C^k$ $n$-manifold and $x \in X$ a point and let $f_1, \ldots, f_m \in C^k(U)$ be functions defined in an open neighborhood $U$ of $x$ such that their differentials at $x$ are linearly independent. Show that there exists a chart at $x$ whose first $m$ components are the $f_i$. 