March 3, 2004 Lecturer Dmitri Zaitsev Hilary Term 2004

Course 425 2003-04

Sheet 7

Due: after the lecture in the beginning of the next term

Exercise 1

Let $F\colon \mathbb{R}^3 \to \mathbb{R}^3$ be defined by $F(x,y,z) = (xy, yz, zx)$. Calculate

$$F^*(dx), \quad F^*(ydx + xdy), \quad F^*(zdx \wedge dy), \quad F^*(dx \wedge dy \wedge dz).$$

Exercise 2

Define the Möbius band and show that it is not orientable.

Exercise 3

(a) Show that the product of two orientable manifolds is orientable.

(b) Let M be the Möbius band. Is $M \times \mathbb{R}$ orientable?

Exercise 4

In the following cases describe an orientation of M (in terms of coordinate charts or nonvanishing forms) and calculate the integral $\int_M \omega$ with respect to this orientation:

(a) $M = (0,1) \subset \mathbb{R}$, $\omega = xdx$.

(b) $M = S^1 \subset \mathbb{R}^2$, $\omega = dx_1 + dx_2$.

(c) $M = S^2 \subset \mathbb{R}^3$, $\omega = dx_1 \wedge dx_2$.

(d) $M = S^2 \subset \mathbb{R}^3$, $\omega = \frac{dx_1 \wedge dx_2}{x_3}$.

Exercise 5

Let ω be a 1-form on S^2. Assume that ω is invariant under rotations $\varphi \in SO(3)$, i.e. $\varphi^*\omega = \omega$. Prove that $\omega = 0$. Is the same conclusion true for a 2-form?