

Course 425 2003-04**S h e e t 6**

Due: after the lecture on Monday, February 9

Exercise 1

Find all integral curves and write a formula for the flow for the vector fields:

(a) $X = \frac{\partial}{\partial x_1} + 2\frac{\partial}{\partial x_2}$

(b) $X = \frac{\partial}{\partial x_1} - \frac{\partial}{\partial x_2}$

Exercise 2

Compute the Lie bracket of the vector fields:

(a) $2x^2 \frac{\partial}{\partial x}$ and $(x+1) \frac{\partial}{\partial x}$ on \mathbb{R} ;

(b) $x_2 \frac{\partial}{\partial x_1}$ and $\frac{\partial}{\partial x_2}$ on \mathbb{R}^2 ;

Exercise 3

Denote by S^2 the standard sphere in \mathbb{R}^3 .

(a) Construct a nonzero vector field on S^2 ;

(b) Construct a vector field on S^2 with only two zeroes;

(c) Construct a vector field on S^2 with only one zero.