November 6, 2003 Lecturer Dmitri Zaitsev Michaelmas Term 2003

Course 425 2003-04

Sheet 3

Due: after the lecture next Thursday

Exercise 1

Given two C^k manifolds M_1 and M_2, prove that the canonical projections $\pi_i \colon M_1 \times M_2 \to M_i$, $i = 1, 2$, are C^k submersions.

Exercise 2

Show that being submanifold is a transitive relation: If M_1 is a submanifold of M_2 and M_2 of M_3, then M_1 is a submanifold of M_3.

Exercise 3

Show: If $S \subset M$ is a submanifold, the inclusion $\imath \colon S \to M$ is an embedding. Vice versa, if S and M are two manifolds and $\imath \colon S \to M$ is an embedding, then the image $\imath(S)$ is a submanifold of M.

Exercise 4

For each $k = 0, 1, \ldots$, define $\varphi_k \colon \mathbb{C} \to \mathbb{C}$ and $\psi_k \colon \mathbb{C}^* \to \mathbb{C}$ ($\mathbb{C}^* := \mathbb{C} \setminus \{0\}$) by $\varphi_k(z) = \psi_k(z) := z^k$. For which k are φ_k and ψ_k immersions, submersions or embeddings?

Exercise 5

Prove that an injective immersion of a compact Hausdorff manifold is an embedding. Can "compact" be dropped?