October 23, 2003 Lecturer Dmitri Zaitsev Michaelmas Term 2003

Course 425 2003-04

Sheet 2

Due: after the lecture next Thursday

Exercise 1

Give an atlas on the unit circle $S^1 \subset \mathbb{R}^2$ consisting only of 2 different pairs $(U_\alpha, \varphi_\alpha)$.

Exercise 2

Show that the atlas consisting of the map $\varphi(x) = x^3$ defines a differentiable structure on $\mathbb{R}$ which is different from the standard one. That is, construct a chart for one atlas which is not a chart for the other atlas.

Exercise 3

Check that the manifold topology defined on a manifold M with an atlas $(U_\alpha, \varphi_\alpha)_{\alpha \in A}$ is the only one for which every map φ_α is a homeomorphism onto its image $\varphi_\alpha(U_\alpha)$.

Exercise 4

Give an explicit atlas on the set

$$M := \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1^2 + x_2^2 = x_3^2 + x_4^2 = 1\}$$

that makes M into a manifold. What is the dimension of this manifold? What is the smoothness class?