Course 425 2003-04

Sheet 1

Due: after the lecture next Thursday

Exercise 1

Which of the following maps $f\colon \mathbb{R} \to \mathbb{R}^2$ define regular arcs and of what class C^k?

$$f(t) = (t + t^2, t - t^2), \quad f(t) = (0, |t|), \quad f(t) = (t, t|t|), \quad f(t) = (\cos(e^t), \sin(e^t)).$$

Exercise 2

Show that the circle cannot be parametrized by an injective regular arc.

Exercise 3

A *surface of revolution* is obtained by rotating a regular plane curve C parametrized by $t \mapsto (x(t), z(t))$ (called the *meridian* or *profile curve*) in the (x, z)-plane around the z-axis in $\mathbb{R}^3$, where C is assumed not to intersect the z-axis (i.e. $x(t) \neq 0$ for all t). It admits a parametrization of the form

$$f(t, \varphi) := \big(x(t)\cos\varphi, x(t)\sin\varphi, z(t)\big).$$

Show that the map f defines a regular surface element.

Exercise 4

A *ruled surface* is obtained by moving a line in $\mathbb{R}^3$ and admits a parametrization of the form

$$f(t, u) := p(u) + tv(u) \in \mathbb{R}^3, \quad t \in \mathbb{R}, \quad u \in I,$$

where I is an interval in $\mathbb{R}$, $p, v\colon I \to \mathbb{R}^3$ are C^1 maps with v nowhere vanishing. Show that f defines a regular surface element provided the vectors $p'(u)$, $v(u)$ and $v'(u)$ are linearly independent for every $u \in I$.