Exercise 1
Which of the following maps $f: \mathbb{R} \to \mathbb{R}^2$ define regular arcs and of what class $C^k$?

$$f(t) = (t + t^2, t - t^2), \quad f(t) = (0, |t|), \quad f(t) = (t, t|t|), \quad f(t) = (\cos(e^t), \sin(e^t)).$$

Exercise 2
Show that the circle cannot be parametrized by an injective regular arc.

Exercise 3
A surface of revolution is obtained by rotating a regular plane curve $C$ parametrized by $t \mapsto (x(t), z(t))$ (called the meridian or profile curve) in the $(x, z)$-plane around the $z$-axis in $\mathbb{R}^3$, where $C$ is assumed not to intersect the $z$-axis (i.e. $x(t) \neq 0$ for all $t$). It admits a parametrization of the form

$$f(t, \varphi) := (x(t)\cos\varphi, x(t)\sin\varphi, z(t)).$$

Show that the map $f$ defines a regular surface element.

Exercise 4
A ruled surface is obtained by moving a line in $\mathbb{R}^3$ and admits a parametrization of the form

$$f(t, u) := p(u) + tv(u) \in \mathbb{R}^3, \quad t \in \mathbb{R}, \quad u \in I,$$

where $I$ is an interval in $\mathbb{R}$, $p, v: I \to \mathbb{R}^3$ are $C^1$ maps with $v$ nowhere vanishing. Show that $f$ defines a regular surface element provided the vectors $p'(u), v(u)$ and $v'(u)$ are linearly independent for every $u \in I$. 