Course 414 2005-06

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Due: after the lecture next Friday

Exercise 1

Prove that

$$\text{Im}(iz) = \text{Re}z, \quad \text{Re}(iz) = -\text{Im}z, \quad |\text{Re}z| + |\text{Im}z| \leq \sqrt{2}|z|.$$

Exercise 2

Sketch the set of points give by the condition:

(i) $1 < |z| < 2$;

(ii) $1 < |z + i| < 2$;

(iii) $\text{Re}((1 + i)\bar{z}) = 2$;

Exercise 3

Find $\arg z$ and $\text{Arg}\, z$ for

(i) $z = i$;

(ii) $z = 1 - i$;

(iii) $z = 2/(1 + \sqrt{3}i)$.

Exercise 4

(i) Show that $\arg(z_1 z_2) = \arg z_1 + \arg z_2$ as sets.

(ii) Show that $\text{Arg}(z_1 z_2) = \text{Arg}\, z_1 + \text{Arg}\, z_2$ provided $-\pi < \text{Arg}\, z_1 + \text{Arg}\, z_2 \leq \pi$.

(iii) Give an example of z_1, z_2 with $\text{Arg}(z_1 z_2) \neq \text{Arg}\, z_1 + \text{Arg}\, z_2$.