November 7 Lecturer Dmitri Zaitsev Michaelmas Term 2019

MAU34205 2019

Sheet 2

Due: after the lecture 15 November

Exercise 1

Determine the zero order of f at 0:

(i) $f(z) = 2z\,\mathsf{cos}z - \mathsf{sin}z - z$;

(ii) $f(z) = (\mathsf{Log}(\mathsf{cos}z + z - \mathsf{sin}\,z))^3$;

(iii) $f(z) = (1 + z^2 - e^{z^3})^{10}$.

Exercise 2

Let (f_n) and (g_n) be compactly convergent sequences of holomorphic functions in Ω.

(i) Show that the sequences (f_n+g_n), $(f_n g_n)$ and $(\mathsf{sin} f_n)$ are also compactly convergent in Ω.

(ii) Is the same conclusion true with "compactly" replaced by "uniformly"?

(iii) Suppose in addition that g_n has no zeros in Ω for each n. Is the sequence f_n/g_n always compactly convergent in Ω?

Exercise 3

Give examples of a connected open set (domain) $\Omega \subset \mathbb{C}$ and a holomorphic function f in Ω such that:

(i) Ω is bounded and $f \not\equiv 0$ has infinitely many zeros;

(ii) the same as in (i) but f is, in addition, bounded on Ω;

Exercise 4

Use Rouché's theorem to find the number of zeroes of the function inside the circle $|z| = 1$:

(i) $f(z) = z^{16} - 19z^5 + z^2 - 2z + e^{z^2}$;

(ii) $f(z) = 2z^{48} - 6z^{73} + z^2 - 48z^{11} + 1 + e^z\mathsf{sin}z$;