Exercise 1
Show that
\[ \frac{e^{i\theta_1} - 1}{e^{i\theta_2} - 1} \]
cannot be not real for all \( \theta_1 \) and \( \theta_2 \).

Exercise 2
Consider the arcs
\[ \gamma_1(t) = t, \quad \gamma_2(t) = it, \quad \gamma_3(t) = (1 + i)t, \quad 0 \leq t \leq 1. \]
Assume that \( f \) is \( \mathbb{R} \)-differentiable at 0 with \( df|_0 \) invertible and \( f \) preserves angles between \( \gamma_1, \gamma_2 \) and between \( \gamma_1, \gamma_3 \). Show that \( f \) satisfies the Cauchy-Riemann equations at 0.

Exercise 3
Prove the differentiation rules for the formal derivatives:

[ (i)] The Leibnitz Rule:
\[ (fg)_z = f_z g + fg_z, \quad (fg)_\bar{z} = f_\bar{z} g + f g_\bar{z}. \]

[ (ii)] The Chain Rule:
\[ (f \circ g)_z = f_w g_z + f_w g_\bar{z}, \quad (f \circ g)_\bar{z} = f_w g_z + f_w g_\bar{z}. \]

Exercise 4
Determine whether the function \( f \) is holomorphic by calculating \( f_z \) using formulas from the previous exercise:

(i) \( f(z) = \cos(z^2\bar{z}^3) \);
(ii) \( f(z) = e^{z^2 + \bar{z}^2} \);
(iii) \( f(z) = \sin(\bar{z}^6) \)