October 11, 2013 Lecturer Dmitri Zaitsev Michaelmas Term 2013

Course 3423/4 2013-14

Sheet 1

Due: after the lecture next Wednesday

Exercise 1

Show that

$$\frac{e^{i\theta_1} - 1}{e^{i\theta_2} - 1}$$

cannot be not real for all θ_1 and θ_2.

Exercise 2

Consider the arcs

$$\gamma_1(t) = t, \quad \gamma_2(t) = it, \quad \gamma_3(t) = (1+i)t, \quad 0 \le t \le 1.$$

Assume that f is $\mathbb{R}$-differentiable at 0 with $df|_0$ invertible and f preserves angles between γ_1, γ_2 and between γ_1, γ_3. Show that f satisfies the Cauchy-Riemann equations at 0.

Exercise 3

Prove the differentiation rules for the formal derivatives:

[(i)] The Leibnitz Rule:

$$(fg)_z = f_z g + f g_z, \quad (fg)_{\bar z} = f_{\bar z} g + f g_{\bar z}.$$

[(ii)] The Chain Rule:

$$(f \circ g)_z = f_w g_z + f_{\bar w} \bar g_z, \quad (f \circ g)_{\bar z} = f_w g_{\bar z} + f_{\bar w} \bar g_{\bar z}$$

Exercise 4

Determine whether the function f is holomorphic by calculating $f_{\bar z}$ using formulas from the previous exercise:

(i) $f(z) = \cos(z^2 \bar z^3)$;

(ii) $f(z) = e^{z^2 + \bar z^2}$;

(iii) $f(z) = \overline{\sin(\bar z^6)}$