Exercise 1
Let $f$ have a pole of order $m$ at a point $z_0$ and $g$ have pole of order $n$ at the same point.
   (i) Does $f + g$ always have an isolated singularity at $z_0$?
   (ii) Does $f + g$ always have a pole at $z_0$?
   (iii) Same question for $h = fg$?
   (iv) In cases $f + g$ or $fg$ have a pole at $z_0$, what are the possible pole orders?

Exercise 2
If $f$ and $g$ are entire functions (holomorphic in $\mathbb{C}$) and $|f(z)| \leq |g(z)|^3$ for all $z$, show that $f(z) = cg(z)^3$ for some constant $c$.

Exercise 3
Suppose that $f$ is meromorphic in $\mathbb{C}$ and bounded outside a disk $B_R(0)$. Show that $f$ is rational. (Hint. Try to eliminate the poles inside the disk.)

Exercise 4
Use Rouché’s theorem to find the number of zeroes of the function inside the circle $|z| = 1$:
   (i) $f(z) = z^{11} - 8z^7 + z^2 - 2z$;
   (ii) $f(z) = 2z^{48} - 2z^3 + z^2 - 18z^5 + 1$;

Exercise 5
Find all Möbius transformations
   (i) sending $1, -1, 0$ onto $0, 1, \infty$ respectively;
   (ii) preserving the imaginary line $i\mathbb{R}$;
   (iii) preserving the unit circle.
Exercise 6

Let $\Omega \subset \mathbb{C}$ be open and connected and $f: \Omega \to \mathbb{C}$ a holomorphic map. Show that either $f \equiv \infty$ or the set $f^{-1}(\infty)$ has no limit points in $\Omega$.

Exercise 7

Find all biholomorphic self-maps of $\Omega$:

(i) $\Omega = \{z : |\text{Im} z| < 1\}$ (Hint. Find a biholomorphic map of the strip onto the upper-half plane using $e^z$.)

(ii) $\Omega = \mathbb{H} \setminus \{2i\}$, where $\mathbb{H}$ is the upper-half plane.