December 7, 2011 Lecturer Dmitri Zaitsev Michaelmas Term 2011

**Course 3423/4 2011-12**

Sheet 3

Due: after the lecture first week of the next term

## Exercise 1

Determine the Laurent series expansion of the function $f$ at $a$ and its ring of convergence:

(i) $f(z) = \frac{z}{z-1}$, $a = 1$;

(ii) $f(z) = (z^2+1)^{-1}$, $a = -i$;

(iii) $f(z) = (z-\pi)^{-3}\cos z$, $a = \pi$;

(iv) $f(z) = \frac{\mathrm{Log}\, z}{(z-i)^3}$, $a = i$;

## Exercise 2

Let $f(z) = \sum_{n=-\infty}^{\infty} a_n(z-z_0)^n$ and $g(z) = \sum_{n=-\infty}^{\infty} b_n(z-z_0)^n$ be Laurent series converging in a ring $r < |z-z_0| < R$. Find the formula for the Laurent series expansion of the product $fg$ and show that it converges in the same ring.

## Exercise 3

Determine the zero order of $f$ at $z_0$:

(i) $f(z) = z\cos z - z$, $z_0 = 0$;

(ii) $f(z) = (\mathrm{Log}(1+z-\sin z))^2$, $z_0 = 2\pi$.

(iii) $f(z) = (1+z^2-e^{z^2})^{20}$, $z_0 = 0$.

## Exercise 4

Determine the type of singularity (removable, pole, essential or not isolated):

(i) $f(z) = \frac{\sin z}{z-\pi}$ at $z_0 = \pi$;

(ii) $f(z) = \frac{\cos z - 1}{z^2}$ at $z_0 = 0$;

(iii) $f(z) = z^4 e^{1/z}$ at $z_0 = 0$;

(iv) $f(z) = \frac{z^2}{e^{1/z}-1}$ at $z_0 = 0$;

## Exercise 5

For each function $f$ from the previous exercise, determine a maximal open set $\Omega \subset \mathbb{C}$ such that $f$ is meromorphic in $\Omega$.