Course 3423/4 2011-12

Sheet 1

Exercise 1

Find θ_1 and θ_2 such that the number

$$\frac{e^{i\theta_1} - 1}{e^{i\theta_2} - 1}$$

is not real.

Exercise 2

Prove the differentiation rules for the formal derivatives:

[(i)] The Leibnitz Rule:

$$(fg)_z = f_z g + fg_z, \quad (fg)_{\overline{z}} = f_{\overline{z}} g + fg_{\overline{z}}.$$

[(ii)] The Chain Rule:

$$(f \circ g)_z = f_w g_z + f_{\bar{w}} \bar{g}_z, \quad (f \circ g)_{\bar{z}} = f_w g_{\bar{z}} + f_{\bar{w}} \bar{g}_{\bar{z}}$$

Exercise 3

Determine whether the function f is holomorphic by calculating $f_{\bar{z}}$ using formulas from the previous exercise:

(i)
$$f(z) = \cos(z\overline{z}^5);$$

(ii) $f(z) = e^{z^2 + \overline{z}};$
(iii) $f(z) = \overline{\sin(\overline{z}^2)}$