Exercise 1

Which of the following sets of vectors are bases for the corresponding space $\mathbb{R}^n$? (The dimension $n$ should be clear from the length of vectors.)

(i) $(1, 1)$;
(ii) $(0, 1), (1, -1)$;
(iii) $(2, 2), (1, 1)$;
(iv) $(-2, 2), (2, 2), (1, 1)$;
(v) $(1, 0, 0, 0), (0, 1, -2, 3), (-1, 3, 2, 1)$;
(vi) $(-1, 0, 1), (0, 3, 0), (1, -1, -1)$.

Exercise 2

Find the coordinates of the vector $v$ with respect to the basis $v_1, \ldots, v_n$ (i.e. the coefficients $k_1, \ldots, k_n$ in the representation $v = k_1v_1 + \cdots + k_nv_n$):

(i) $v = (-2, 3), v_1 = (-2, 1), v_2 = (-1, 2)$;
(ii) $v = (2, -1, 4), v_1 = (-1, 1, 0), v_2 = (1, 1, 0), v_3 = (1, -1, 1)$;
(iii) $v = (-1, 2, 1, 2), v_1 = (1, 0, 0, 0), v_2 = (1, 1, 0, 0), v_3 = (0, 0, 1, 0), v_4 = (1, 1, 1, 1)$.