March 5, 2008 Lecturer Dmitri Zaitsev Hilary Term 2008

Course 2E2 2007-08 (SF Engineers & MSISS & MEMS)

S h e e t 12

Due: at the end of the tutorial

Exercise 1

Find the smallest period for the functions:

$$\sin 2x, \quad \cos\frac{x}{7}, \quad \sin n\pi x, \quad |\sin x|.$$

Solution

(i) $\sin 2(x+\pi) = \sin 2x$, hence π is a period. The value 1 is attained at $x = \pi/4$ and $x = \pi + (\pi/4)$ and nowhere else in the interval $(\pi/4, \pi + (\pi/4))$. Hence π is the smallest period;

(ii) $\cos\frac{x+14\pi}{7} = \cos\frac{x}{7}$, hence 14π is a period. The value 1 is attained at $x = 0$ and $x = 14\pi$ and nowhere else in the interval $(0, 14\pi)$. Hence 14π is the smallest period;

(iii) $\sin n\pi(x + \frac{2}{n}) = \sin n\pi x$, hence $2/n$ is a period. The value 1 is attained at $x = \frac{1}{2n}$ and $x = \frac{1}{2n} + \frac{2}{n}$ but nowhere else in the interval $(\frac{1}{2n}, \frac{1}{2n} + \frac{2}{n})$. Hence $2/n$ is the smallest period;

(iv) $|\sin(x + \pi)| = |-\sin x| = |\sin x|$, hence π is a period. The value 1 is attained at $x = \pi/2$ and $x = 3\pi/2$ and nowhere else in the interval $(\pi/2, 3\pi/2)$. Hence π is the smallest period.

Exercise 2

Which systems of functions are orthogonal with respect to the inner product $\langle f, g\rangle = \int_{-\pi}^{\pi} f(x)g(x)dx$?

(i) $\{1, x\}$;

(ii) $\{1, (x^2 - \frac{\pi^2}{3})\}$;

(iii) $\{1, x, (x^2 - \frac{\pi^2}{3})\}$.

Solution

(i) One has $\langle 1, x\rangle = \int_{-\pi}^{\pi} x\,dx = 0$, hence the system is orthogonal;

(ii) One has $\langle 1, (x^2 - \frac{\pi^2}{3})\rangle = \int_{-\pi}^{\pi}(x^2 - \frac{\pi^2}{3})\,dx = (\frac{x^3}{3} - \frac{\pi^2 x}{3})\Big|_{-\pi}^{\pi} = 0$, hence the system is orthogonal;

(iii) One has $\langle 1, x\rangle = \langle 1, (x^2 - \frac{\pi^2}{3})\rangle = 0$ but

$$\left\langle x, (x^2 - \frac{\pi^2}{3})\right\rangle = \int_{-\pi}^{\pi}(x^3 - \frac{\pi^2 x}{3})\,dx = (\frac{x^4}{4} - \frac{\pi^2 x^2}{6})\Big|_{-\pi}^{\pi} \neq 0.$$

Hence the system is not orthogonal.