Course 2E1 2006-07 (SF Engineers & MSISS & MEMS)

S h e e t 16

Due: at the end of the tutorial

Exercise 2(ii)

Find an equation for the plane spanned by the vectors: $\mathbf{u} = (1, -1, 1)$, $\mathbf{v} = (-1, 0, 1)$;

Solution The span is the set of all linear combinations of $\mathbf{u}$ and $\mathbf{v}$, i.e. the set of all expressions $k_1\mathbf{u} + k_2\mathbf{v}$, hence the set of all vectors (x, y, z) that can be written as $k_1\mathbf{u}+k_2\mathbf{v}$, hence the set of all (x, y, z) for which the following vector equation is solvable in (k_1, k_2):

$$k_1 \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + k_2 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}.$$

This vector equation is equivalent to the system

$$\begin{cases} k_1 - k_2 = x \\ -k_1 = y \\ k_1 + k_2 = z \end{cases}$$

from which we can eliminate both k_1 and then k_2:

$$\begin{cases} k_1 = -y \\ k_2 = k_1 - x \\ -y + k_2 = z \end{cases}$$

or

$$\begin{cases} k_1 = -y \\ k_2 = -y - x \\ -2y - x = z \end{cases}$$

The system is solvable in (k_1, k_2) precisely when $-2y - x = z$ or

$$x + 2y + z = 0,$$

which is the desired equation.

It is a good idea to check the correctness of this equation by substituting into it $(x, y, z) = \mathbf{u} = (1, -1, 1)$ and $(x, y, z) = \mathbf{v} = (-1, 0, 1)$ (the given vectors):

$$1 + 2 \cdot (-1) + 1 = 0, \quad (-1) + 2 \cdot 0 + 1 = 0.$$