**Course 2E1 2006-07 (SF Engineers & MSISS & MEMS)**

Due: at the end of the tutorial

**Sheet 14**

**Exercise 1**

(i) $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$;

(ii) $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$;

(iii) product makes no sense;

**Exercise 2**

(i) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$;

(ii) $\begin{pmatrix} 2 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$;

(iii) $\begin{pmatrix} 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & -1 & 1 \\ -1 & 1 & 1 & 1 \end{pmatrix}$;

(iv) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$;

(v) $\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ -1 & 1 & 0 \\ 5 & 0 & 0 \\ 0 & 0 & -4 \end{pmatrix}$.

**Sheet 15**

**Exercise 1**

(i) $(-1, 6)$;

(ii) $(-1, 0)$;

(iii) $\begin{pmatrix} 1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & 1/2 \end{pmatrix} \begin{pmatrix} -3 \\ 1 \end{pmatrix}$;

(iv) $\begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

**Exercise 2**

(i) a subspace;

(ii) not a subspace;

(iii) a subspace.

**Sheet 16**

**Exercise 1**

(i) don't span;

(ii) span;

(iii) span.

**Exercise 2**

(i) $(x, y, z) = (2, 0, -3)t$ or $x = 2t, y = 0, z = -3t$;

(ii) $(x, y, z, u, v, w) = (0, 2, 1, 0, 6, 5)t$ or $x = 0, y = 2t, z = t, u = 0, v = 6t, w = 5t$;

(iii) a possible equation is $x + 2y + z = 0$.

**Sheet 17**

**Exercise 1**

(i) independent;

(ii) dependent;

(iii) dependent;

(iv) independent;

(v) dependent.

**Exercise 2**

(i) not a basis;

(ii) a basis;

(iii) not a basis;

(iv) not a basis;

(v) not a basis;

(vi) a basis.

**Sheet 18**

**Exercise 1**

(i) $c_1 = \frac{9}{5}$, $c_2 = -\frac{8}{5}$;

(ii) $c_1 = -\frac{3}{2}$, $c_2 = 2$, $c_3 = -\frac{3}{2}$;

(iii) $c_1 = -1$, $c_2 = 0$, $c_3 = -\frac{1}{2}$, $c_4 = 2$.

**Exercise 2**

(i) Row space: possible basis $\{\mathbf{r}_1\}$, $\mathbf{r}_1 = (1, \frac{1}{2})$, dimension 1;

Column space: possible basis $\{\mathbf{c}_1\}$, $\mathbf{c}_1 = (2)$, dimension 1;

Null space: possible basis $\{\mathbf{v}_1\}$, $\mathbf{v}_1 = \begin{pmatrix} -\frac{1}{2} \\ 1 \end{pmatrix}$, dimension 1.

(ii) Row space: possible basis $\{\mathbf{r}_1\}$, $\mathbf{r}_1 = (1)$, dimension 1;

Column space: possible basis $\{\mathbf{c}_1\}$, $\mathbf{c}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, dimension 1;

Null space: empty basis, dimension 0.

(iii) Row space: possible basis $\{\mathbf{r}_1\}$, $\mathbf{r}_1 = (1, -2)$, dimension 1;

Column space: possible basis $\{\mathbf{c}_1\}$, $\mathbf{c}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, dimension 1;

Null space: possible basis $\{\mathbf{v}_1\}$, $\mathbf{v}_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, dimension 1.

(iv) Row space: possible basis $\{\mathbf{r}_1, \mathbf{r}_2\}$, $\mathbf{r}_1 = (1, -2, 0)$, $\mathbf{r}_2 = (0, 0, 1)$, dimension 2;

Column space: possible basis $\{\mathbf{c}_1, \mathbf{c}_2\}$, $\mathbf{c}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, $\mathbf{c}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, dimension 2;

Null space: possible basis $\{\mathbf{v}_1\}$, $\mathbf{v}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, dimension 1.

(v) Row space: possible basis $\{\mathbf{r}_1, \mathbf{r}_2\}$, $\mathbf{r}_1 = (1, -2)$, $\mathbf{r}_2 = (0, 1)$, dimension 2;

Column space: possible basis $\{\mathbf{c}_1, \mathbf{c}_2\}$, $\mathbf{c}_1 = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$, $\mathbf{c}_2 = \begin{pmatrix} -2 \\ 2 \\ 0 \end{pmatrix}$, dimension 2;

Null space: empty basis, dimension 0.

**Sheet 20**

**Exercise 1**

(i) $\mathbf{v}_1 = (3, 4)$, $\mathbf{v}_2 = (\frac{96}{72}, -\frac{72}{25})$;

(ii) $\mathbf{v}_1 = (1, 1, 0)$, $\mathbf{v}_2 = (\frac{1}{2}, -\frac{1}{2}, 1)$, $\mathbf{v}_3 = (1, -1, -1)$;

(iii) $\mathbf{v}_1 = (1, 1, 0, 0)$, $\mathbf{v}_2 = (\frac{1}{2}, -\frac{1}{2}, 1, 0)$, $\mathbf{v}_3 = (1, -1, -1, 0)$, $\mathbf{v}_3 = (0, 0, 0, 1)$;

**Sheet 21**

**Exercise 1**

(i) $x = -\frac{1}{5}$;

(ii) $x = \frac{5}{7}$, $y = -\frac{8}{7}$;

(iii) $x = \frac{3}{4}$, $y = -\frac{5}{4}$, $z = \frac{3}{4}$.

**Exercise 2**

(i) $(\lambda - 1)(\lambda + 2)$;

(ii) $\lambda^2 + 26$;

(iii) $\lambda(\lambda - 1)(\lambda + 3)$;

(iv) $\lambda(\lambda^2 - 3)$.

**Sheet 22**

**Exercise 1**

(i) Eigenvalues: $-1$, $-2$, possible eigenvectors: $\mathbf{v}_1 = (1,1)$ for $\lambda_1 = -1$ and $\mathbf{v}_2 = (0,1)$ for $\lambda_1 = -2$;

(ii) Eigenvalues: 1, 0, 5, possible eigenvectors: $\mathbf{v}_1 = (1,0,0)$ for $\lambda_1 = 1$, $\mathbf{v}_2 = (4,-2,3)$ for $\lambda_2 = 0$ and $\mathbf{v}_3 = (3,4,4)$ for $\lambda_3 = 5$.

**Exercise 2**

(i) $P = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$, $D = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}$;

(ii) $P = \begin{pmatrix} 1 & 4 & 3 \\ 0 & -2 & 4 \\ 0 & 3 & 4 \end{pmatrix}$, $D = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 5 \end{pmatrix}$;