Course 2E1 2005-06 (SF Engineers & MSISS & MEMS)

S h e e t 20

Due: in the tutorial sessions next Wednesday/Thursday

Exercise 1

Find the least squares approximate solution of the linear system:

(i) $\begin{cases} x = 2 \\ 2x = -1 \end{cases};$

Solution. Write the system in the matrix form $A\mathbf{u} = \mathbf{b}$:

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} (x) = \begin{pmatrix} 2 \\ -1 \end{pmatrix},$$

so $A = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$. Now pass to the associated normal system

$$A^T A\mathbf{u} = A^T\mathbf{b},$$

i.e.

$$\begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} (x) = \begin{pmatrix} 1 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \end{pmatrix}.$$

Simplifying we have

$$5(x) = 0$$

and the (least squares approximate) solution is $x = 0$.

(ii) $\begin{cases} x + y = 1 \\ x - y = 0 \\ 2x + y = 0 \end{cases};$

Solution. Now the matrix form is

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

The associated normal system is

$$\begin{pmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 1 & 2 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

or

$$\begin{pmatrix} 6 & 2 \\ 2 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$

Now we just need to solve this system, i.e.

$$\begin{cases} 6x + 2y = 1 \\ 2x + 3y = 1 \end{cases}$$

and the result will be the needed solution.

(iii) $$\begin{cases} x = 1 \\ y = 1 \\ z = 1 \\ x + y + z = 0 \end{cases}.$$

Solution.

Again, the matrix form is

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix},$$

the associated normal system is

$$\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix},$$

i.e.

$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Note that the coefficient matrix is symmetric. Now the system can be solved the usual way and the result gives the solution to the problem.