Course 2E1 2005-06 (SF Engineers & MSISS & MEMS)

S h e e t 17

Due: in the tutorial sessions next Wednesday/Thursday

Exercise 1

Exercise 2

Find bases and dimensions for the row, column and null spaces of the matrix:

(i) $\begin{pmatrix} 2 & -1 \end{pmatrix}$;

(ii) $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$;

(iii) $\begin{pmatrix} 1 & -3 \\ -1 & 3 \end{pmatrix}$;

(iv) $\begin{pmatrix} 1 & -3 & 0 \\ -1 & 3 & 1 \end{pmatrix}$;

(v) $\begin{pmatrix} 1 & -3 \\ -1 & 3 \\ 2 & 0 \end{pmatrix}$;

See the solution to the sheet 17 of the year 2004-5 for bases and dimensions for the row and column spaces. Here we only look for bases of the null spaces.

(i) $\begin{pmatrix} 2 & -1 \end{pmatrix}$;

Solution.

The null space is the solution space of the system

$$\begin{pmatrix} 2 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

or $2x_1 - x_2 = 0$. We can set $x_1 = t$ with t a free parameter and solve our equation in the form $x_2 = 2x_1 = 2t$. Thus the general solution is

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} t \\ 2t \end{pmatrix} = t \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

and we have a possible basis consisting of one vector $\{(1,2)\}$. The dimension of the null space is the number of vectors in a basis, hence it is 1.

(ii) $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$;

Solution.
The null space is the solution space of the system

$$\begin{pmatrix} 2 \\ -1 \end{pmatrix} (x) = 0.$$

Note that the size of the column (x) is matching the horizontal size of the matrix, so the matrix multiplication makes sense. The system gives $x = 0$. Here no variable is free, the space is 0-dimensional and any basis is empty.

(iii) $\begin{pmatrix} 1 & -3 \\ -1 & 3 \end{pmatrix}$;

Solution.
The null space is the solution space of the system

$$\begin{pmatrix} 1 & -3 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0,$$

which can be solved as $x_2 = t$, $x_1 = 3t$. Hence the general solution is

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 3t \\ t \end{pmatrix} = t \begin{pmatrix} 3 \\ 1 \end{pmatrix},$$

a basis is $\{(3,1)\}$ and the dimension is 1.

(iv) $\begin{pmatrix} 1 & -3 & 0 \\ -1 & 3 & 1 \end{pmatrix}$;

Solution.
The null space is the solution space of the system

$$\begin{pmatrix} 1 & -3 & 0 \\ -1 & 3 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0.$$

This time the variable vector has 3 components as needed for the matrix multiplication. The general solution is

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3t \\ t \\ 0 \end{pmatrix} = t \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix},$$

hence a basis is $\{(3, 1, 0)\}$ and the dimension is again 1.

(v) $\begin{pmatrix} 1 & -3 \\ -1 & 3 \\ 2 & 0 \end{pmatrix}$;

Solution.

Now the null space is the solution space of the system

$$\begin{pmatrix} 1 & -3 \\ -1 & 3 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 0$$

and the only solution is $x_1 = x_2 = 0$. This means the space is 0-dimensional and any basis is empty.