Course 2E1 2004-05 (SF Engineers & MSISS & MEMS)

S h e e t 3

Due: in the tutorial sessions next Wednesday/Thursday

Exercise 1

Calculate first order partial derivatives $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$:

(i) $f(x, y) = 12x - 2y$,

(ii) $f(x, y) = x^2 + y^2$,

(iii) $f(x, y) = e^{x+y}$,

(iv) $f(x, y) = x/y$.

Solution

(i) $f(x, y) = 12x - 2y$: $\frac{\partial f}{\partial x} = 12$, $\frac{\partial f}{\partial y} = -2$;

(ii) $f(x, y) = x^2 + y^2$: $\frac{\partial f}{\partial x} = 2x$, $\frac{\partial f}{\partial y} = 2y$;

(iii) $f(x, y) = e^{x+y}$: $\frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} = e^{x+y}$;

(iv) $f(x, y) = x/y$: $\frac{\partial f}{\partial x} = \frac{1}{y}$, $\frac{\partial f}{\partial y} = -\frac{x}{y^2}$.

Exercise 2

Find the partial derivative $\frac{\partial f}{\partial x}$ for a function $z = f(x, y)$ defined implicitly by the equation:

(i) $z + z^3 = x - y$,

(ii) $xyz = e^z$.

Solution of (i)

Substituting $z = f(x, y)$ we obtain the equation for $f(x, y)$:

$$f(x, y) + (f(x, y))^3 = x - y.$$

Taking partial derivatives of both sides in x we have

$$\frac{\partial f}{\partial x}(x, y) + 3(f(x, y))^2 \frac{\partial f}{\partial x}(x, y) = 1$$

from which we can find $\frac{\partial f}{\partial x}$ as

$$\frac{\partial f}{\partial x} = \frac{1}{1+3(f(x,y))^2} = \frac{1}{1+3z^2}.$$

Solution of (ii)
Analogously substitute $z = f(x,y)$:

$$xy\, f(x,y) = e^{f(x,y)}.$$

Now differentiate both sides in x:

$$y\, f(x,y) + xy\frac{\partial f}{\partial x}(x,y) = e^{f(x,y)}\frac{\partial f}{\partial x}(x,y).$$

Finally find $\frac{\partial f}{\partial x}$:

$$\frac{\partial f}{\partial x} = \frac{y\, f(x,y)}{e^{f(x,y)} - xy} = \frac{yz}{e^z - xy}.$$

Exercise 3

Calculate higher order partial derivatives f_{xy} and f_{xxx} of the following functions:

(i) $f(x,y) = (xy)^2 + \frac{\sin x}{e^x}$,
(ii) $f(x,y) = \cos x + y \sin x$.

Solution

(i) $f(x,y) = (xy)^2 + \frac{\sin x}{e^x}$: $f_{xy} = (f_x)_y = (f_y)_x = (2x^2y)_x = 4xy,\ f_{xxx} = 2\frac{\cos x + \sin x}{e^x}$.
(ii) $f(x,y) = \cos x + y\sin x$: $f_{xy} = \cos x,\ f_{xxx} = \sin x - y\cos x$.