

Course 2E1 2004-05 (SF Engineers & MSISS & MEMS)

S h e e t 2

Due: in the tutorial sessions next Wednesday/Thursday

Exercise 1

Find the function's domain and range:

- (i) $f(x, y) = x - y$,
- (ii) $f(x, y) = x^2 + y^2$,
- (iii) $f(x, y) = \sqrt{x^2 - y^2}$,
- (iv) $f(x, y) = \sin(x + y)$,
- (v) $f(x, y) = \sqrt{-x}$,
- (vi) $f(x, y) = x/y$.

Note that a function $f(x, y)$ may not explicitly depend on some of the variables.

Solution

- (i) $z = x - y$, domain: $-\infty < x < \infty$, $-\infty < y < \infty$; range: $-\infty < z < \infty$.
- (ii) $z = x^2 + y^2$, domain: $-\infty < x < \infty$, $-\infty < y < \infty$; range: $z \geq 0$,
- (iii) $z = \sqrt{x^2 - y^2}$, domain $|x| \geq |y|$; range: $z \geq 0$,
- (iv) $z = \sin(x + y)$, domain: $-\infty < x < \infty$, $-\infty < y < \infty$; range: $-1 \leq z \leq 1$,
- (v) $z = \sqrt{-x}$, domain: $x \leq 0$, $-\infty < y < \infty$; range: $z \geq 0$,
- (vi) $f(x, y) = x/y$, domain: $-\infty < x < \infty$, $y \neq 0$; range: $-\infty < z < \infty$.

Exercise 2

Describe level curves of the functions in Exercise 1.

Solution

- (i) $f(x, y) = x - y$: parallel lines $y = x + \text{const}$.
- (ii) $f(x, y) = x^2 + y^2$: circles centered at $(0, 0)$,
- (iii) $f(x, y) = \sqrt{x^2 - y^2}$: hyperbolas with asymptotes $y = \pm x$,
- (iv) $f(x, y) = \sin(x + y)$: parallel lines $y = -x + \text{const}$,
- (v) $f(x, y) = \sqrt{-x}$: parallel lines $x = \text{const}$ with $\text{const} \geq 0$,
- (vi) $f(x, y) = x/y$: lines passing through the origin without the origin

$$x = \text{const} \cdot y, \quad y \neq 0.$$

Exercise 3

Find domain and range and describe level surfaces for the following functions of 3 variables:

- (i) $f(x, y, z) = x + y + z$,
- (ii) $f(x, y, z) = x^2$,
- (iii) $f(x, y, z) = \sin(x^2 + y^2 + z^2)$.

Solution

- (i) $w = x + y + z$, domain: $-\infty < x < \infty$, $-\infty < y < \infty$, $-\infty < z < \infty$; range: $-\infty < w < \infty$; the level surfaces are the planes $x + y + z = \text{const}$.
- (ii) $w = x^2$, domain: $-\infty < x < \infty$, $-\infty < y < \infty$, $-\infty < z < \infty$; range: $w \geq 0$; the level surfaces are the planes $x = \text{const}$.
- (iii) $w = \sin(x^2 + y^2 + z^2)$, $-\infty < x < \infty$, $-\infty < y < \infty$, $-\infty < z < \infty$; range: $-1 \leq w \leq 1$; the level surfaces are the spheres $x^2 + y^2 + z^2 = \text{const}$.

Exercise 4

Find limits:

$$\lim_{(x,y) \rightarrow (0,1)} \frac{x+y}{\cos x}, \quad \lim_{(x,y) \rightarrow (0,0)} \frac{5x^2 + 5y^2}{\sqrt{x^2 + y^2}}, \quad \lim_{(x,y,z) \rightarrow (5,0,3)} \sqrt{x^2 + y^2 - z^2}.$$

Solution: 1, 0, 4.