Exercise 1

Find the function’s domain and range:

(i) $f(x, y) = x - y$,
(ii) $f(x, y) = x^2 + y^2$,
(iii) $f(x, y) = \sqrt{x^2 - y^2}$,
(iv) $f(x, y) = \sin(x + y)$,
(v) $f(x, y) = \sqrt{-x}$,
(vi) $f(x, y) = x/y$.

Note that a function $f(x, y)$ may not explicitly depend on some of the variables.

Exercise 2

Describe level curves of the functions in Exercise 1.

Exercise 3

Find domain and range and describe level surfaces for the following functions of 3 variables:

(i) $f(x, y, z) = x + y + z$,
(ii) $f(x, y, z) = x^2$,
(iii) $f(x, y, z) = \sin(x^2 + y^2 + z^2)$.

Exercise 4

Find limits:

$$\lim_{(x, y)\rightarrow (0, 1)} \frac{x + y}{\cos x}, \quad \lim_{(x, y)\rightarrow (0, 0)} \frac{5x^2 + 5y^2}{\sqrt{x^2 + y^2}}, \quad \lim_{(x, y, z)\rightarrow (5, 0, 3)} \sqrt{x^2 + y^2 - z^2}.$$