April 5, 2005 Lecturer Dmitri Zaitsev Hilary Term 2005

Course 2E1 2004-05 (SF Engineers & MSISS & MEMS)

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Due: in the tutorial sessions first Wednesday/Thursday in the next term

Exercise 1

Calculate the length of $\mathbf{u} = (1, -1, 1)$, the distance between $\mathbf{u}$ and $\mathbf{v} = (0, 1, 1)$ and the angle between $\mathbf{u}$ and $\mathbf{v}$

(i) with respect to the (standard) Euclidean inner product;

(ii) with respect to the inner product given by $\langle \mathbf{u}, \mathbf{v} \rangle = 2u_1v_1 + 3u_2v_2 + u_3v_3$.

Exercise 2

Which of the following bases are orthogonal and which are orthonormal?

(i) $(1, 0)$, $(0, 2)$;

(ii) $(1, 0, 1)$, $(1, 1, -1)$, $(-1, 0, 1)$;

(iii) $(1, 0, 0)$, $(0, \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}})$, $(0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$;

Exercise 3

Calculate the coordinates of $\mathbf{v}$ relative to the basis in Exercise 2 (iii):

(i) $\mathbf{v} = (1, 1, 1)$;

(ii) $\mathbf{v} = (-1, 1, -1)$.

Exercise 4

Use the Gram-Schmidt process to transform $\mathbf{u}_1 = (1, 1, 1)$, $\mathbf{u}_2 = (1, 1, 0)$, $\mathbf{u}_3 = (1, 0, 0)$ into an orthogonal basis.