March 1, 2005 Lecturer Dmitri Zaitsev Hilary Term 2005

Course 2E1 2004-05 (SF Engineers & MSISS & MEMS)

S h e e t 16

Due: in the tutorial sessions next Wednesday/Thursday

Exercise 1

Find the coordinates of the vector $\mathbf{v}$ with respect to the basis $\mathbf{v}_1, \ldots, \mathbf{v}_n$ (i.e. the coefficients $c_1, \ldots, c_n$ in the representation $\mathbf{v} = c_1\mathbf{v}_1 + \cdots + c_n\mathbf{v}_n$:

(i) $\mathbf{v} = (3, -7)$, $\mathbf{v}_1 = (1, -1)$, $\mathbf{v}_2 = (1, 1)$;

(ii) $\mathbf{v} = (2, -1, 3)$, $\mathbf{v}_1 = (1, 0, 0)$, $\mathbf{v}_2 = (1, 1, 0)$, $\mathbf{v}_3 = (1, 1, 1)$;

(iii) $\mathbf{v} = (1, 1, 1, 1)$, $\mathbf{v}_1 = (1, 0, 1, 0)$, $\mathbf{v}_2 = (1, 1, 0, 0)$, $\mathbf{v}_3 = (0, 1, 1, 0)$, $\mathbf{v}_4 = (-1, 1, 1, 1)$.

Exercise 2

Find the vector form $\mathbf{x} = \mathbf{x}_0 + c_1\mathbf{v}_1 + \cdots + c_n\mathbf{v}_n$ (i.e. find vectors $\mathbf{x}_0$, $\mathbf{v}_1, \ldots, \mathbf{v}_n$) for the general solution solution of the system:

(i)

$$\begin{cases} x_1 - 3x_2 + x_3 = 1 \\ x_2 = 2 \end{cases}$$

(ii)

$$\begin{cases} x_1 + x_2 + 2x_3 = 5 \\ x_1 + x_3 - x_4 = -2 \\ 2x_1 + x_2 + 3x_3 - x_4 = 3 \end{cases}$$

Exercise 3

Find a basis for the nullspace of A:

(i)

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix}$$

(ii)

$$A = \begin{pmatrix} 2 & 0 & -1 & 0 \\ 1 & -1 & 1 & -1 \\ 3 & -1 & 0 & -1 \\ 1 & 1 & -2 & 1 \end{pmatrix}$$