Exercise 1

Find the standard matrix for the linear transformations $T$ defined by the equations

(i) $w_1 = x_1 - x_2$, $w_2 = x_1 + x_2$,
(ii) $w_1 = 2x - z$, $w_2 = y$, $w_3 = z$,
(iii) $w_1 = x_1$, $w_2 = x_1 + x_2$, $w_3 = x_1 + x_2 + x_3$, $w_4 = x_1 + x_2 + x_3 + x_4$,
and by the formulas

(v) $T(x_1, x_2) = (x_1, -x_2)$,
(iv) $T(x_1, x_2, x_3) = (x_2, -x_1, x_2 + x_1, 3x_3, -4x_3)$.

Exercise 2

Find $T(x) = Ax$ for the matrix $A$ and the vector $x$ whenever the product makes sense (i.e. the sizes of $A$ and $x$ fit together):

(i) $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $x = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$,
(ii) $A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 2 & 3 \end{pmatrix}$, $x = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$,
(iii) $A = \begin{pmatrix} -1 & 1 & 0 \\ 1 & 0 & -1 \end{pmatrix}$, $x = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

Exercise 3

Use matrix multiplication to find:

(i) the reflection of the vector $(1, -2)$ about the $y$-axis;
(ii) the orthogonal projection of the vector $(-1, 2)$ to the $x$-axis;
(iii) the image of the vector $(-1, 1)$ under rotation through the angle $\frac{\pi}{3}$ about the origin.
(iv) the image of the vector $(-2, 7)$ under rotation through the angle $-\frac{\pi}{4}$ about the origin.