Course 2E02 2015 (SF Engineers & MSISS & MEMS)

S h e e t 4

Due: at the end of the tutorial

Exercise 1

Which of the following sets of vectors are linearly dependent?

(i) $(0, 1)$, $(-1, -2)$;

(ii) $(0, -1, 1)$, $(1, 1, 0)$, $(-2, 0, -2)$;

(iii) $(0, 0, -1, 0, 0)$, $(1, 2, 2, 1, 1)$, $(1, 2, 0, 1, 1)$.

Exercise 2

Which of the following sets of vectors are bases for the corresponding space $\mathbb{R}^n$? (The dimension n should be clear from the length of vectors.)

(i) $(-1, 2)$;

(ii) $(0, 1)$, $(1, -2)$;

(iii) $(-2, -2)$, $(3, 3)$;

(iv) $(1, -1)$, $(5, -12)$, $(-1, 1)$;

(v) $(1, 1, 2, 0)$, $(-1, 1, 5, 3)$, $(1, 3, 2, 1)$;

(vi) $(1, 0, 1)$, $(0, 1, 0)$, $(2, 1, -2)$.

Exercise 3

Find the coordinates of the vector $\mathbf{v}$ with respect to the basis $\mathbf{v}_1, \ldots, \mathbf{v}_n$ (i.e. the coefficients $k_1, \ldots, k_n$ in the representation $\mathbf{v} = k_1\mathbf{v}_1 + \cdots + k_n\mathbf{v}_n$:

(i) $\mathbf{v} = (2, -1)$, $\mathbf{v}_1 = (1, 1)$, $\mathbf{v}_2 = (-1, 2)$;

(ii) $\mathbf{v} = (1, 0, 1, -1)$, $\mathbf{v}_1 = (1, 0, 1, 0)$, $\mathbf{v}_2 = (1, 1, 0, 0)$, $\mathbf{v}_3 = (0, 0, 1, 0)$, $\mathbf{v}_4 = (1, 0, 0, 1)$.