Exercise 1

Find the matrix for the linear transformations $T$ defined by the equations

(i) $w_1 = x_1, \quad w_2 = -x_2 + x_1,$
(ii) $w_1 = x, \quad w_2 = y - 2z, \quad w_3 = -z,$
(iii) $w_1 = x_4, \quad w_2 = x_4 + x_3, \quad w_3 = x_4 - x_3 - x_2, \quad w_4 = x_4 + x_3 + x_2 - x_1,$

and by the formulas
(iv) $T(x_1, x_2) = (-x_1, x_2),$
(v) $T(x_1, x_2, x_3) = (x_3, x_1 - x_2, x_1 + 4x_2 + x_3, -2x_2, x_3, x_1).$

Exercise 2

Find $T(x) = Ax$ for the matrix $A$ and the vector $x$ whenever the product makes sense (i.e. the sizes of $A$ and $x$ fit together):

(i) $A = \begin{pmatrix} 0 & -1 \\ 1 & -2 \end{pmatrix}, \quad x = \begin{pmatrix} 1 \\ -2 \end{pmatrix},$
(ii) $A = \begin{pmatrix} 0 & 1 & -1 \\ 4 & 0 & -2 \end{pmatrix}, \quad x = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix},$
(iii) $A = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 4 & 1 \end{pmatrix}, \quad x = \begin{pmatrix} -5 \\ 4 \end{pmatrix}.$

Exercise 3

Use matrix multiplication to find:

(i) the reflection of the vector $(2, -1)$ about the $x$-axis;
(ii) the orthogonal projection of the vector $(2, -1)$ to the $y$-axis;
(iii) the image of the vector $(2, -1)$ under rotation through the angle $\frac{\pi}{4}$ about the origin.
(v) the image of the vector $(2, 1, -1)$ under rotation through the angle $\frac{\pi}{6}$ about $z$-axis.