

Course 2E02 2014 (SF Engineers & MSISS & MEMS)**S h e e t 2**

Due: at the end of the tutorial

Exercise 1

Find the matrix for the linear transformations T defined by the equations

- (i) $w_1 = x_1, \quad w_2 = -x_2 + x_1,$
- (ii) $w_1 = x, \quad w_2 = y - 2z, \quad w_3 = -z,$
- (iii) $w_1 = x_4, \quad w_2 = x_4 + x_3, \quad w_3 = x_4 - x_3 - x_2, \quad w_4 = x_4 + x_3 + x_2 - x_1,$
and by the formulas
- (iv) $T(x_1, x_2) = (-x_1, x_2),$
- (v) $T(x_1, x_2, x_3) = (x_3, x_1 - x_2, x_1 + 4x_2 + x_3, -2x_2, x_3, x_1).$

Exercise 2

Find $T(\mathbf{x}) = A\mathbf{x}$ for the matrix A and the vector \mathbf{x} whenever the product makes sense (i.e. the sizes of A and \mathbf{x} fit together):

- (i) $A = \begin{pmatrix} 0 & -1 \\ 1 & -2 \end{pmatrix}, \mathbf{x} = \begin{pmatrix} 1 \\ -2 \end{pmatrix},$
- (ii) $A = \begin{pmatrix} 0 & 1 & -1 \\ 4 & 0 & -2 \end{pmatrix}, \mathbf{x} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix},$
- (iii) $A = \begin{pmatrix} 1 & 2 & 0 \\ -1 & 4 & 1 \end{pmatrix}, \mathbf{x} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}.$

Exercise 3

Use matrix multiplication to find:

- (i) the reflection of the vector $(2, -1)$ about the x -axis;
- (ii) the orthogonal projection of the vector $(2, -1)$ to the y -axis;
- (iii) the image of the vector $(2, -1)$ under rotation through the angle $\frac{\pi}{4}$ about the origin.
- (v) the image of the vector $(2, 1, -1)$ under rotation through the angle $\frac{\pi}{6}$ about z -axis.