

Course 2E02 2013 (SF Engineers & MSISS & MEMS)**S h e e t 5**

Due: at the end of the tutorial

Exercise 1

Find the rank and the nullity of the matrix:

(i) $\begin{pmatrix} 3 & -3 & 3 \\ -2 & 2 & -2 \end{pmatrix};$

(ii) $\begin{pmatrix} 2 & -2 & -1 \\ 1 & 1 & 1 \\ 1 & -3 & -2 \end{pmatrix}.$

Exercise 2

Calculate the length of $\mathbf{u} = (1, 1, 0)$, the distance between \mathbf{u} and $\mathbf{v} = (0, 1, 0)$ and the angle between \mathbf{u} and \mathbf{v}

- (i) with respect to the standard dot product;
- (ii) with respect to the inner product given by $\langle \mathbf{u}, \mathbf{v} \rangle = u_1 v_1 + 2u_2 v_2 + 3u_3 v_3$.

Exercise 3

Which of the following bases are orthogonal and which are orthonormal (with respect to the standard dot product)?

- (i) $(1, 0), (0, -2);$
- (ii) $(0, 0, -1), (1, -1, 0), (-1, -1, 0);$
- (iii) $(1, 0, 0), (0, -\frac{3}{5}, -\frac{4}{5}), (0, \frac{4}{5}, -\frac{3}{5});$