Course 2E02 2013 (SF Engineers & MSISS & MEMS)

S h e e t   4

Due: at the end of the tutorial

Exercise 1

Which of the following sets of vectors are bases for the corresponding space $\mathbb{R}^n$? (The dimension n should be clear from the length of vectors.)

   (i) $(1, -2)$;
  (ii) $(0, 1)$, $(1, -2)$;
 (iii) $(-4, 4)$, $(3, -3)$;
 (iv) $(1, 4)$, $(5, -12)$, $(1, 1)$;
  (v) $(1, 1, 6, 0)$, $(11, 1, 5, 3)$, $(1, 3, 2, 1)$;
 (vi) $(1, 0, 1)$, $(0, 1, 0)$, $(-2, 1, -2)$.

Exercise 2

Find the coordinates of the vector $\mathbf{v}$ with respect to the basis $\mathbf{v}_1, \ldots, \mathbf{v}_n$ (i.e. the coefficients $k_1, \ldots, k_n$ in the representation $\mathbf{v} = k_1 \mathbf{v}_1 + \cdots + k_n \mathbf{v}_n$:

   (i) $\mathbf{v} = (1, -2)$, $\mathbf{v}_1 = (1, 1)$, $\mathbf{v}_2 = (-1, 2)$;
  (ii) $\mathbf{v} = (1, 0, 2, -2)$, $\mathbf{v}_1 = (1, 0, 1, 0)$, $\mathbf{v}_2 = (1, 1, 0, 0)$, $\mathbf{v}_3 = (0, 0, 1, 0)$, $\mathbf{v}_4 = (1, 0, 0, 1)$.

Exercise 3

Find bases and dimensions for the row, column and null spaces of the matrix:

   (i) $\begin{pmatrix} 1 & 2 & 0 \\ 1 & -2 & 1 \end{pmatrix}$;

  (ii) $\begin{pmatrix} -1 & 2 \\ -3 & 6 \\ 4 & -8 \end{pmatrix}$.