January 28, 2013 Lecturer Dmitri Zaitsev Hilary Term 2013

Course 2E02 2013 (SF Engineers & MSISS & MEMS)

Sheet 2

Due: at the end of the tutorial

Exercise 1

Find the matrix for the linear transformations T defined by the equations

(i) $w_1 = x_1, \quad w_2 = x_2 - x_1,$

(ii) $w_1 = x, \quad w_2 = y + 2z, \quad w_3 = -z,$

(iii) $w_1 = x_4, \quad w_2 = x_4 - x_3, \quad w_3 = x_4 + x_3 - x_2, \quad w_4 = x_4 + x_3 + x_2 - x_1,$

and by the formulas

(iv) $T(x_1, x_2) = (-x_1, -x_2),$

(v) $T(x_1, x_2, x_3) = (x_3, x_1 - x_2, x_1 + 4x_2 + x_3, -2x_2, 5x_3).$

Exercise 2

Find $T(\mathbf{x}) = A\mathbf{x}$ for the matrix A and the vector $\mathbf{x}$ whenever the product makes sense (i.e. the sizes of A and $\mathbf{x}$ fit together):

(i) $A = \begin{pmatrix} 0 & -1 \\ -1 & -2 \end{pmatrix}$, $\mathbf{x} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}$,

(ii) $A = \begin{pmatrix} 0 & 1 & 1 \\ 4 & 0 & -2 \end{pmatrix}$, $\mathbf{x} = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$,

(iii) $A = \begin{pmatrix} 1 & -2 & 0 \\ -1 & 14 & 1 \end{pmatrix}$, $\mathbf{x} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$.

Exercise 3

Use matrix multiplication to find:

(i) the reflection of the vector $(2, 1)$ about the x-axis;

(ii) the orthogonal projection of the vector $(2, 1)$ to the y-axis;

(iii) the image of the vector $(2, 1)$ under rotation through the angle $\frac{\pi}{3}$ about the origin.

(v) the image of the vector $(2, 1, -3)$ under rotation through the angle $\frac{\pi}{3}$ about z-axis.