

**Course 2E02 2011 (SF Engineers & MSISS & MEMS)****S h e e t 5**

---

Due: at the end of the tutorial

---

**Exercise 1**

Find the rank and the nullity of the matrix:

- (i)  $\begin{pmatrix} 2 & -1 & -1 \end{pmatrix}$ ;
- (ii)  $\begin{pmatrix} 2 & -1 & -1 \\ 1 & 1 & 1 \\ 1 & -2 & -2 \end{pmatrix}$ .

**Exercise 2**

Calculate the length of  $\mathbf{u} = (-1, 0, 1)$ , the distance between  $\mathbf{u}$  and  $\mathbf{v} = (1, 1, 0)$  and the angle between  $\mathbf{u}$  and  $\mathbf{v}$

- (i) with respect to the standard dot product;
- (ii) with respect to the inner product given by  $\langle \mathbf{u}, \mathbf{v} \rangle = 3u_1v_1 + u_2v_2 + 2u_3v_3$ .

**Exercise 3**

Which of the following bases are orthogonal and which are orthonormal (with respect to the standard dot product)?

- (i)  $(1, 0), (0, -2)$ ;
- (ii)  $(0, 0, -2), (1, -1, 0), (1, 1, 0)$ ;
- (iii)  $(1, 0, 0), (0, -\frac{3}{5}, -\frac{4}{5}), (0, \frac{4}{5}, -\frac{3}{5})$ ;