Exercise 1

Find the rank and the nullity of the matrix:

(i) \[
\begin{pmatrix}
2 & -1 & -1 \\
-1 & -1 & -1 \\
-1 & -1 & -1 \\
\end{pmatrix}
\]

(ii) \[
\begin{pmatrix}
2 & -1 & -1 \\
1 & 1 & 1 \\
1 & -2 & -2 \\
\end{pmatrix}
\]

Exercise 2

Calculate the length of \( \mathbf{u} = (-1, 0, 1) \), the distance between \( \mathbf{u} \) and \( \mathbf{v} = (1, 1, 0) \) and the angle between \( \mathbf{u} \) and \( \mathbf{v} \)

(i) with respect to the standard dot product;

(ii) with respect to the inner product given by \( \langle \mathbf{u}, \mathbf{v} \rangle = 3u_1v_1 + u_2v_2 + 2u_3v_3 \).

Exercise 3

Which of the following bases are orthogonal and which are orthonormal (with respect to the standard dot product)?

(i) \((1, 0), (0, -2)\);

(ii) \((0, 0, -2), (1, -1, 0), (1, 1, 0)\);

(iii) \((1, 0, 0), (0, -\frac{2}{3}, -\frac{1}{3}), (0, \frac{4}{9}, -\frac{2}{9})\);