MAU22E01 2020 (SF Engineers & MSISS & MEMS)

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Pra	actice sheet - will not be marked

It is important to be able to do all the problems, including unmarked ones, to ensure you are prepared for the exam.

Exercise 1

Find the eigenvalues and corresponding eigenvectors of the following matrix:

(i)

$$A = \begin{pmatrix} 3 & -1 & 0 \\ 3 & -1 & 0 \\ -1 & -1 & 2 \end{pmatrix},$$

(ii)

$$A = \begin{pmatrix} 3 & -1 & 0 \\ 3 & -1 & 0 \\ 1 & -1 & 2 \end{pmatrix},$$

(iii)

$$A = \begin{pmatrix} 3 & -1 & 0 \\ 3 & -1 & 0 \\ 1 & -1 & 1 \end{pmatrix}.$$

Exercise 2

Find an invertible matrix P and a diagonal matrix D diagonalizing A, i.e. satisfying $P^{-1}AP = D$, where A is as in Exercise 1(ii) and (iii). Is a diagonalization possible for the matrix A in Exercise 1(i)?

Exercise 3

For the system of ordinary differential equations

$$\begin{pmatrix} y_1' \\ y_2' \\ y_3' \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix},$$

where A is as in Exercise 1(ii):

- (i) find a general solution;
- (ii) find the solution y satisfying the initial value condition

$$y(0) = (1, 0, -1).$$

Exercise 4

For A is as in Exercise 1(ii) and (iii), compute the power A^{50} .