November 16, 2020 Lecturer Dmitri Zaitsev

MAU22E01 2020 (SF Engineers & MSISS & MEMS)

S h e e t 6

Practice sheet - will not be marked

It is important to be able to do all the problems, including unmarked ones, to ensure you are prepared for the exam.

Exercise 1

Calculate the length of $\mathbf{u} = (1, 0, 1)$, the distance between $\mathbf{u}$ and $\mathbf{v} = (0, 1, -1)$ and the angle between $\mathbf{u}$ and $\mathbf{v}$

(i) relative to the standard dot product;

(ii) relative to the inner product given by $\langle \mathbf{u}, \mathbf{v} \rangle = u_1 v_1 + 2u_2 v_2 + 2u_3 v_3$.

Exercise 2

Which of the following bases are orthogonal and which are orthonormal relative to the dot product?

(i) $(-1, 1)$, $(0, 2)$;

(ii) $(0, 0, 1)$, $(2, -2, 0)$, $(1, 1, 0)$;

(iii) $(1, 0, 0)$, $(0, \frac{3}{5}, \frac{4}{5})$, $(0, \frac{4}{5}, -\frac{3}{5})$.

Exercise 3

Calculate the coordinates of $\mathbf{v} = (3, -2, -1)$ relative to the orthogonal basis

$$\{(-2, 0, 0), (0, 2, 3), (0, 3, -2)\} :$$

(i) relative to the standard dot product;

(ii) relative to the inner product $\langle \mathbf{u}, \mathbf{v} \rangle = 4u_1 v_1 + u_2 v_2 + u_3 v_3$ (check that the given basis is still orthogonal relative to this inner product).

Exercise 4

Find the orthogonal projection of the vector $\mathbf{v}$ onto the plane spanned by the orthogonal basis $\{\mathbf{u}_1, \mathbf{u}_2\}$ (relative to the dot product), where

$$\mathbf{u}_1 = (-1, 2, 0), \quad \mathbf{u}_2 = (2, 1, -1),$$

and

(i) $\mathbf{v} = (1, 0, 1)$;

(ii) $\mathbf{v} = (1, -1, 1)$.