October 19, 2020 Lecturer Dmitri Zaitsev

MAU22E01 2020 (SF Engineers & MSISS & MEMS)

S h e e t 3

Practice sheet - will not be marked

It is important to be able to do all the problems, including unmarked ones, to ensure you are prepared for the exam.

Exercise 1

Find a system of implicit equations for the span of the vectors:

(i) $\mathbf{u} = (1, 1, -2), \quad \mathbf{v} = (1, -3, 0);$

(ii) $\mathbf{u} = (1, 1, -2, 0), \quad \mathbf{v} = (1, -3, 0, 1), \quad \mathbf{w} = (0, 4, -2, -1).$

Exercise 2

Which of the following sets of vectors are linearly dependent?

(i) $(1, 1)$, $(-1, 0)$;

(ii) $(0, 1, 1)$, $(1, 1, 0)$, $(2, 0, -2)$;

(iii) $(0, -4, 0, 0, -2)$, $(0, 2, 3, 1, 1)$, $(0, 2, 0, 0, 1)$.

Exercise 3

Which of the following sets of vectors are bases for the corresponding space $\mathbb{R}^n$? (The dimension n should be clear from the length of vectors.)

(i) $(1, -1)$;

(ii) $(1, 0)$, $(1, 1)$;

(iii) $(-1, -1)$, $(2, 2)$;

(iv) $(1, -1)$, $(15, 22)$, $(-1, 1)$;

(v) $(1, -1, 2, 1)$, $(1, 1, 5, -3)$, $(1, 1, 2, 1)$;

(vi) $(1, 0, 1)$, $(1, 1, 0)$, $(-1, 1, 0)$.

Exercise 4

Find the coordinates of the vector $\mathbf{v}$ with respect to the basis $\mathbf{v}_1, \dots, \mathbf{v}_n$ (i.e. the coefficients $k_1, \dots, k_n$ in the representation $\mathbf{v} = k_1\mathbf{v}_1 + \cdots + k_n\mathbf{v}_n$):

(i) $\mathbf{v} = (-2, 1)$, $\mathbf{v}_1 = (1, -1)$, $\mathbf{v}_2 = (1, 1)$;

(ii) $\mathbf{v} = (1, -3, 2)$, $\mathbf{v}_1 = (1, 1, 0)$, $\mathbf{v}_2 = (1, 0, 1)$, $\mathbf{v}_3 = (0, 1, 1)$;