October 5, 2020 Lecturer Dmitri Zaitsev

MAU22E01 2020 (SF Engineers & MSISS & MEMS)

S h e e t 1

Practice sheet - will not be marked

It is important to be able to do all the problems, including unmarked ones, to ensure you are prepared for the exam.

Exercise 1

Find $\mathbf{v}+\mathbf{u}$, $3\mathbf{u}$, the length $\|\mathbf{u}\|$, $\|\mathbf{v}\|$, the dot product $\mathbf{u}\cdot\mathbf{v}$, the angle between $\mathbf{u}$ and $\mathbf{v}$ and determine whether $\mathbf{u}$ and $\mathbf{v}$ are orthogonal (or for which values of parameters $\mathbf{u}$ and $\mathbf{v}$ are orthogonal, if any are present):

(i) $\mathbf{u}=(0,2)$, $\mathbf{v}=(2,1)$;

(ii) $\mathbf{u}=(-3,k,0,k)$, $\mathbf{v}=(0,3k,-1,3)$.

(iii) $\mathbf{u}=(1,0,0,-1,0,1)$, $\mathbf{v}=(0,2,0,0,-k,k)$;

Exercise 2

Write the system in the matrix form:

(i)

$$\begin{cases} z-2z-2y &= 0 \\ y+x &= 3 \end{cases}$$

(ii)

$$\begin{cases} 2z-4t+x-4y &= -1 \\ 2y &= 0 \\ z-t &= -3 \end{cases}$$

Exercise 3

Find the (standard) matrix of the linear transformations T defined by the equations

(i) $w_1=-x_1, \quad w_2=x_2-x_1+x_3,$

(ii) $w_1=x-y+z, \quad w_2=z+y, \quad w_3=-x, \quad w_4=x,$
and by the formula

(iii) $T(x_1,x_2,x_3,x_4)=(0,x_1,x_3-x_2,x_1-2x_4+x_3,0)$.

Exercise 4

Find $T(\mathbf{x})=A\mathbf{x}$ for the matrix A and the vector $\mathbf{x}$ whenever the product makes sense (i.e. the dimensions of A and $\mathbf{x}$ fit together):

(i) $A=\begin{pmatrix} 0 & -1 & 1 & -1 \\ 1 & -1 & 5 & 0 \end{pmatrix}$, $\mathbf{x}=\begin{pmatrix} 12 \\ -2 \end{pmatrix}$,

(ii) $A=\begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & -2 \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$, $\mathbf{x}=\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$,

(iii) $A=\begin{pmatrix} 1 & 2 \\ -1 & 1 \\ 0 & -1 \\ -1 & 0 \end{pmatrix}$, $\mathbf{x}=\begin{pmatrix} 2 \\ -1 \end{pmatrix}$.