November 19, 2018 Lecturer Dmitri Zaitsev

MAU22E01 2019 (SF Engineers & MSISS & MEMS)

Sheet 9

Due: at the end of the tutorial

Exercise 1

For the matrix

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 3 & -2 \\ 0 & -6 & 4 \end{pmatrix},$$

(i) Find the eigenvalues and corresponding eigenvectors.

(ii) Find an invertible matrix P and a diagonal matrix D diagonalizing A, i.e. satisfying $P^{-1}AP = D$.

Exercise 2

Use Exercise 1 to solve (i.e. find a general solution of) the system of ordinary differential equations

$$\begin{pmatrix} y_1' \\ y_2' \\ y_3' \end{pmatrix} = A \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix},$$

where A is as in Exercise 2. Hint. Diagonalize the matrix A to obtained a decoupled system of ordinary differential equations, then use the general solution $u'(t) = Ce^{at}$ for an equation $u' = au$, where a is any constant.

Exercise 3

Find solutions of the system in Exercise 2 satisfying the initial value problem

$$(y_1(0), y_2(0), y_3(0)) = (0, 0, 1).$$