Exercise 1

For the matrix

\[
A = \begin{pmatrix}
1 & 2 & -1 \\
0 & 3 & -2 \\
0 & -6 & 4
\end{pmatrix},
\]

(i) Find the eigenvalues and corresponding eigenvectors.

(ii) Find an invertible matrix \(P\) and a diagonal matrix \(D\) diagonalizing \(A\), i.e. satisfying \(P^{-1}AP = D\).

Exercise 2

Use Exercise 1 to solve (i.e. find a general solution of) the system of ordinary differential equations

\[
\begin{pmatrix}
y'_1 \\
y'_2 \\
y'_3
\end{pmatrix} = A \begin{pmatrix}
y_1 \\
y_2 \\
y_3
\end{pmatrix},
\]

where \(A\) is as in Exercise 2. Hint. Diagonalize the matrix \(A\) to obtained a decoupled system of ordinary differential equations, then use the general solution \(u'(t) = Ce^{at}\) for an equation \(u' = au\), where \(a\) is any constant.

Exercise 3

Find solutions of the system in Exercise 2 satisfying the initial value problem

\((y_1(0), y_2(0), y_3(0)) = (0, 0, 1)\).