Exercise 1

Find the orthogonal projection of the vector \( \mathbf{v} \) onto the plane spanned by the orthogonal basis \( \{ \mathbf{u}_1, \mathbf{u}_2 \} \) (with respect to the standard dot product), where

\[
\mathbf{u}_1 = (1, 3, 0), \quad \mathbf{u}_2 = (-3, 1, -1),
\]

and

(i) \( \mathbf{v} = (1, 0, -1) \);
(ii) \( \mathbf{v} = (1, 1, -1) \).

Exercise 2

Use the Gram-Schmidt process to transform the given basis into orthogonal one:

(i) \( \mathbf{u}_1 = (-1, 0), \mathbf{u}_2 = (2, -3) \);
(ii) \( \mathbf{u}_1 = (1, 0, -1), \mathbf{u}_2 = (1, 0, 0), \mathbf{u}_3 = (2, -1, 0) \).

Exercise 3

Find the characteristic polynomials of the following matrices:

(i) \[
\begin{pmatrix}
1 & -2 \\
0 & -1
\end{pmatrix};
\]
(ii) \[
\begin{pmatrix}
0 & -2 \\
1 & 0
\end{pmatrix};
\]
(iii) \[
\begin{pmatrix}
1 & 2 & 1 \\
0 & 3 & -2 \\
0 & 0 & -3
\end{pmatrix};
\]
(iv) \[
\begin{pmatrix}
0 & 2 & -1 \\
1 & 0 & 2 \\
0 & -2 & 1
\end{pmatrix}.
\]