Exercise 1

Calculate the length of \( u = (1, 0, 2) \), the distance between \( u \) and \( v = (0, 1, -1) \) and the angle between \( u \) and \( v \)

(i) with respect to the standard dot product;

(ii) with respect to the inner product given by \( \langle u, v \rangle = u_1v_1 + 4u_2v_2 + u_3v_3 \).

Exercise 2

Determine which expression \( \langle u, v \rangle \) provides an inner product in \( \mathbb{R}^2 \):

(i) \( \langle u, v \rangle = 4u_1v_1 - u_2v_2 \);

(ii) \( \langle u, v \rangle = 2u_1v_1 + u_2v_2 \);

(iii) \( \langle u, v \rangle = u_1^2 + v_1^2 \).

Exercise 3

Which of the following bases are orthogonal with respect to the standard dot product?

(i) \( (1, 4), (-4, 1) \);

(ii) \( (0, 0, -1), (2, -2, 0), (3, 3, 0) \);

(iii) \( (1, 0, 0), (0, \frac{3}{5}, -\frac{4}{5}), (0, \frac{4}{5}, \frac{3}{5}) \).